OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS   **2642**

Probability & Statistics 2

Tuesday   **18 JANUARY 2005**   Afternoon   1 hour 20 minutes

Additional materials:
  Answer booklet
  Graph paper
  List of Formulae (MF8)

TIME   1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

* Write your Name, Centre Number and Candidate Number in the spaces provided on the answer booklet.
* Answer **all** the questions.
* Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
* You are permitted to use a graphic calculator in this paper.

INFORMATION FOR CANDIDATES

* The number of marks is given in brackets [ ] at the end of each question or part question.
* The total number of marks for this paper is 60.
* Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
* **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1   A secretary is typing a document. The number of typing mistakes the secretary makes per page can be modelled by a Poisson distribution with mean 2. Find the probability that

    (i) in a document consisting of one page, the secretary makes more than 3 mistakes,   [2]

    (ii) in a document consisting of two pages, the secretary makes a total of fewer than 6 mistakes. [2]

2   The editor of a local newspaper is attempting to determine what proportion of the adults in the area served by the newspaper is interested in environmental matters. One issue of the newspaper therefore contains a questionnaire which readers are invited to complete and return.

    (i) Give two reasons why the results obtained may be biased.   [2]

    (ii) Describe briefly an unbiased method of obtaining the information.   [3]

3   The lifetime, T months, of properly made tap washers is modelled by a normal distribution with mean μ and standard deviation σ.

    (i) It is given that $P(T > 80.0) = 0.05$ and $P(T < 70.0) = 0.75$. Find the values of μ and σ.   [6]

    (ii) Some tap washers are badly made and therefore have a very short lifetime. Give a reason why a normal distribution may not be a good model for the distribution of the lifetimes of all washers.
                                                                                                         [1]

4   A local government spokesman claims that at least three-quarters of the residents of a town are in favour of plans to build a new by-pass for the town. An opinion poll showed that 10 out of a random sample of 16 residents of the town were in favour of the plans. Test, at the 10% significance level, whether the results of the opinion poll are consistent with the spokesman's claim, stating your hypotheses clearly.
                                                                                                         [7]

5   The random variable Y has a Poisson distribution with mean 20.

    (i) Use the exact distribution to calculate $P(Y = 22)$.   [2]

    (ii) Use a suitable approximation to calculate $P(Y = 22)$.   [4]

    (iii) Calculate the percentage error in the approximate value.   [2]

6   (i) Explain what is meant by a Type I error.   [1]

    (ii) The continuous random variable X has the distribution $N(\mu, \sigma^2)$. A test of the hypothesis $H_0 : \mu = 25$ is carried out at the 5% significance level, once a day for 300 days. Given that on each day the value of μ is 25, use a normal approximation to find the probability that a Type I error is made on at least 20 days.   [6]

    (iii) Explain whether, in answering part (ii), it is necessary to assume that the outcomes of the tests are independent.   [1]

7 Two models are proposed for the continuous random variable X. Model 1 has probability density function

$$f_1(x) = \begin{cases} 1 - \frac{1}{2}x & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$
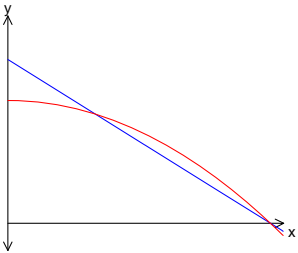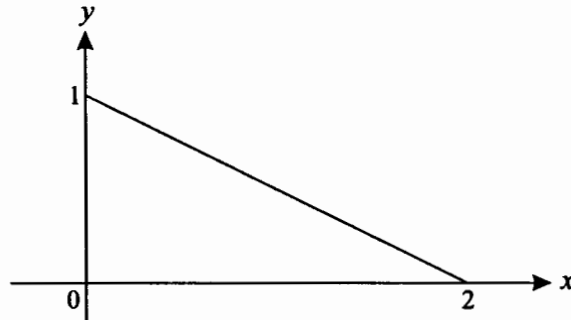

The diagram shows the graph of $y = f_1(x)$.

(i) Find the upper quartile of X (i.e., find the value q such that $P(X < q) = 0.75$) according to model 1. [4]

Model 2 has probability density function

$$f_2(x) = \begin{cases} k(4 - x^2) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) The graph of $y = f_2(x)$ intersects the y-axis at the point $(0, 4k)$. Copy the diagram showing the graph of $y = f_1(x)$. On your copy sketch the graph of $y = f_2(x)$, explaining how you can tell without doing any integration that $4k < 1$. [4]

(iii) State whether the value of q obtained from model 1 is greater than, equal to, or less than the value given by model 2. Use your diagram to justify your answer. [2]

8 **(i)** A random variable X has the distribution $N(\mu, \sigma^2)$. The mean of a sample of 5 observations of X is denoted by $\bar{X}$. State the distribution of $\bar{X}$, giving the values of any parameters. [2]

(ii) A group of scientists is attempting to identify subatomic particles called *ocrons*. Ocrons have a mean path length of less than 42 cm. The path lengths of a random sample of five particles thought to be ocrons are measured, and the mean path length of the sample is found to be 36.6 cm. Path lengths are known to be normally distributed random variables with standard deviation 8 cm. Carry out a test, at the 10% significance level, of whether the population mean path length is less than 42 cm, stating your hypotheses clearly. [7]

(iii) A second group of scientists carries out a test that is identical, except that they use a 5% significance level. If the mean observed path length of the particles is consistent with a population mean of less than 42 cm, the scientists will claim that the particles are ocrons. State what the use of this smaller significance level suggests about the intentions of the scientists in deciding whether or not to claim that the observed particles are ocrons. [2]

1	(i)	$1 - 0.8571$	M1		0.8571 seen, $1 - 0.6767$, or formula, ± 1 term
		0.143	A1	**2**	Answer, a.r.t. 0.143 [0.1429]
	(ii)	$\mu = 4$	M1		$\mu = 4$ stated or implied
		0.785	A1	**2**	Answer, a.r.t. 0.785 [0.7851]
2	(i)	Those with strong feelings are more likely to respond	B1		One valid relevant reason, e.g. "not everyone will respond"
		Only readers will respond	B1	**2**	A different valid relevant reason
	(ii)	Obtain a list of the population	B1		*Any 3 out of:* List population [*not* readers]
		Given each member a number	B1		Allocate numbers to list [*not* random numbers]
		Select using random numbers	B1	**3**	Random numbers mentioned (not "hat")
		Contact those selected			Contact those selected
					[SR: Census: B1B0B1]
3	(i)	$\dfrac{80 - \mu}{\sigma} = \Phi^{-1}(0.95) = 1.645$	M1		Standardise one and equate to Φ^{-1} [*not* Φ]
			A1		Both standardised $= \Phi^{-1}$, signs correct
			B1		Both [1.64, 1.65] and [0.674, 0.675]
		$\dfrac{70 - \mu}{\sigma} = \Phi^{-1}(0.75) = 0.674$	M1		Correct method to solve for one unknown
			A1		σ correct, a.r.t. 10.3
		$\sigma = $ 10.3	A1	**6**	μ correct, a.r.t. 63.1 [63.06]
		$\mu = $ 63.1			[σ^2: M1A0B1M1A0A1]
					[cc: M1A1B1M1A0A0]
	(ii)	Distribution bimodal/not symmetrical so normal distribution not appropriate	B1	**1**	Any relevant valid comment: "not symmetric" is enough
4		H_0: $p = 0.75$	B1		Hypotheses correctly stated in terms of p or π
		H_1: $p < 0.75$	B1		[wrong/no symbol: B1B0, but x or $\bar{x}$: B0]
		B(16, 0.75)	M1		B (16, 0.75) stated or implied
		α: 0.1897, compare with 0.10	M1		Correct comparison
		β: Compare 10 with 9; $P(\leq 9) = 0.0796$	A1		Correct p value, or probability for critical region
		Do not reject H_0	M1		Correct method & comparison
		Insufficient evidence that spokesman is wrong	A1√	**7**	Outcome interpreted in context
					[SR: N(12, 3): B1B1 M1M0A0 M0A0]
					[SR: 2-tail: B1B0 M1M0A1 M1A1]
5	(i)	$e^{-20} \dfrac{20^{22}}{22!}$ $= 0.0769$	M1		Formula correct
			A1	**2**	Answer, a.r.t. 0.0769
	(ii)	N(20, 20)	B1		N(20, 20) seen or implied, allow 20^2 etc
		$\Phi\left(\dfrac{22.5 - 20}{\sqrt{20}}\right) - \Phi\left(\dfrac{21.5 - 20}{\sqrt{20}}\right)$	M1		$\dfrac{x - \lambda}{\sqrt{\lambda}}$ twice, allow wrong or no cc, or no √
		$= 0.7119 - 0.6311$ $= 0.0808$	A1		Both, ccs correct, √λ
			A1	**4**	Answer, in range 0.08055 to 0.08085
					[SR: PDF: $\dfrac{1}{\sqrt{20 \times 2\pi}} \exp -\frac{1}{2}\left(\dfrac{22 - 20}{\sqrt{20}}\right)^2$ M2A2]
	(iii)	$\dfrac{0.0808 - 0.07691}{0.07691} \times 100\% = $ 5.06%	M1		$\pm[$(ii) $-$ (i)$] \div$(i) seen or implied, a.e.f.
			A1	**2**	Answer in range 4.77 to 5.14

6	(i)	Reject H_0 when it is correct	B1	**1**	Correct statement [*not* "probability"]
	(ii)	B(300, 0.05)	M1		B(300, 0.05) stated or implied
		N(15;)	A1		Normal, correct value of μ, stated
		14.25 or 3.77(5)	A1		Correct value of σ^2 or σ
		$\dfrac{19.5-15}{\sqrt{14.25}}$ [= 1.192]	M1		Standardise with np and $\sqrt{npq}$, npq, np or $\sqrt{np}$
			A1		Correct cc and $\sqrt{npq}$
		0.117	A1	**6**	Answer, a.r.t. 0.117
	(iii)	Yes, required by binomial	B1	**1**	"Yes" stated with "binomial" mentioned
7	(i)	$\displaystyle\int_0^q (1-\tfrac{1}{2}x)dx = 0.75$	M1		Equate integral to 0.75
		$q - \dfrac{q^2}{4} = \dfrac{3}{4}$	A1		Correct equation, a.e.f., allow x etc throughout
		$q^2 - 4q + 3 = 0; (q-3)(q-1) = 0$	M1		Solve relevant quadratic
		$q = 1$	A1	**4**	$q = 1$ only, cwd
	(ii)		M1		Curve concave-downwards, positive in $0 \le x \le 2$
			B1		Intersects x-axis at same place as line, curve must be positive in $0 \le x \le 2$
			A1		Decreasing curve, f(0) < 1, clearly crossing straight line
		Total area must be 1 also Hence curve must cross line.	B1	**4**	Area mentioned, argument not wrong
	(iii)	Upper quartile of model 2 is higher as graph of f_2 is above that of f_1	B1 B1	**2**	Correct statement, from reasonable graph Correct justification, from reasonable graph
8	(i)	N($\mu, \sigma^2/5$)	B1 B1	**2**	Normal, mean μ Variance $\sigma^2/5$ [*not* $\sigma/\sqrt{5}$]
	(ii)	H_0: $\mu = 42$	B1		One hypothesis correct, μ used
		H_1: $\mu < 42$	B1		Second hypothesis correct, μ used [wrong/no symbol: B1B0, but x or $\bar{x}$: B0]
	α:	$z = \dfrac{36.6-42}{8/\sqrt{5}}$	M1		α: Correct form for z [allow $42 - 36.6$ unless *explicitly* $\mu = 36.6$]
		$z = -1.51$	A1		Correct z, a.r.t. (–)1.51
		$-1.51 < -1.28$ *or* $\Phi(z)$ [= 0.9345] > 0.90	B1√		-1.28 seen, allow + if consistent, *or* explicit comparison of 0.9345√ with 0.90/0.0655√ with 0.10, p correct to 3 SF
or	β:	CV = $42 - k.8/\sqrt{5}$	(M1)		β: Correct form for CV, must use 42
		$k = 1.28$	(B1)		$k = 1.28$ seen
		CV = 37.4 > 36.6	(A1)		CV = 37.4 and compare 36.6
		Reject H_0	M1		Correct overall method/comparison ($\sqrt{5}$ needed) and "reject"/"do not reject"
		Significant evidence that mean path length is less than 42	A1√	**7**	Correct contextualised conclusion
	(iii)	Lower significance level means decreased probability of rejecting H_0 when true, i.e. of claiming it is an ocron when it is not, so scientists want to be more cautious about accepting particles as ocrons.	M1 A1	**2**	Any valid contextualised comment, e.g. "More accurate" Comment equivalent to last clause on left [NB: P(claim it's an ocron when it's not) smaller P(not claim it's an ocron when it is) bigger]